

SVD Based MIMO
Transmission:

Example:

Consider a 3×3 MIMO channel

$$H = \begin{bmatrix} 2 & -6 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$\underbrace{\quad}_{\bar{c}_1} \quad \underbrace{\quad}_{\bar{c}_2} \quad \underbrace{\quad}_{\bar{c}_3}$

$r=3, t=3$

Columns are orthogonal

$\bar{c}_i^H \bar{c}_j = 0$

$$\bar{C}_1 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} \quad \bar{C}_2 = \begin{bmatrix} -6 \\ 4 \\ 0 \end{bmatrix}$$

$$\bar{C}_1^H \bar{C}_2 = \begin{bmatrix} 2 & 3 & 0 \end{bmatrix} \begin{bmatrix} -6 \\ 4 \\ 0 \end{bmatrix}$$

$$= -12 + 12 + 0$$

$$= 0$$

$$H = \begin{bmatrix} 2 & -6 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Normalize each column

$$\downarrow H = \begin{bmatrix} \frac{2}{\sqrt{13}} & -\frac{6}{\sqrt{52}} & 0 \\ \frac{3}{\sqrt{13}} & \frac{4}{\sqrt{52}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{13} & 0 & 0 \\ 0 & \sqrt{52} & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

U

Singular values
are NOT in
decreasing order

$$\begin{aligned}
 H &= \begin{bmatrix} \frac{2}{\sqrt{13}} & -\frac{6}{\sqrt{52}} & 0 \\ \frac{3}{\sqrt{13}} & \frac{4}{\sqrt{52}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{13} & 0 & 0 \\ 0 & \sqrt{52} & 0 \\ 0 & 0 & 2 \end{bmatrix} \\
 &= \begin{bmatrix} -\frac{6}{\sqrt{52}} & \frac{2}{\sqrt{13}} & 0 \\ \frac{4}{\sqrt{52}} & \frac{3}{\sqrt{13}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & \sqrt{52} & 0 \\ \sqrt{13} & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} \\
 &= \underbrace{\begin{bmatrix} -\frac{6}{\sqrt{52}} & \frac{2}{\sqrt{13}} & 0 \\ \frac{4}{\sqrt{52}} & \frac{3}{\sqrt{13}} & 0 \\ 0 & 0 & 1 \end{bmatrix}}_U \underbrace{\begin{bmatrix} \sqrt{52} & 0 & 0 \\ 0 & \sqrt{13} & 0 \\ 0 & 0 & 2 \end{bmatrix}}_{\Sigma} \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{V^H}
 \end{aligned}$$

$$U^H U = I$$

$$V^H V = V V^H = I$$

$$\Sigma = \begin{bmatrix} \sqrt{52} & 0 & 0 \\ 0 & \sqrt{13} & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\sigma_1 = \sqrt{52} \quad \sigma_2 = \sqrt{13} \quad \sigma_3 = 2$$

$$\sigma_1, \sigma_2, \sigma_3 > 0$$

$$\text{Further } \sigma_1 > \sigma_2 > \sigma_3$$

Decreasing order.

At Receiver:

$$\tilde{\mathbf{y}} = \mathbf{U}^H \mathbf{\bar{y}}$$
$$\underbrace{\begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \tilde{y}_3 \end{bmatrix}}_{\tilde{\mathbf{y}}} = \underbrace{\begin{bmatrix} -\frac{6}{\sqrt{52}} & \frac{4}{\sqrt{52}} & 0 \\ \frac{2}{\sqrt{13}} & \frac{3}{\sqrt{13}} & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{U}^H} \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}}_{\mathbf{\bar{y}}}$$

At the Transmitter

$$\underline{\bar{x}} = V \tilde{\underline{x}}$$

Precoding

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix}$$

$\underline{\bar{x}}$ V $\tilde{\underline{x}}$

Transmit Preprocessing
Precoding.

$$\tilde{y} = \sum \tilde{x} + \tilde{w}$$

$$\begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \tilde{y}_3 \end{bmatrix} = \begin{bmatrix} \sqrt{52} & 0 & 0 \\ 0 & \sqrt{13} & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} + \begin{bmatrix} \tilde{w}_1 \\ \tilde{w}_2 \\ \tilde{w}_3 \end{bmatrix}$$

$$\left. \begin{aligned} \tilde{y}_1 &= \sqrt{52} \tilde{x}_1 + \tilde{w}_1 \\ \tilde{y}_2 &= \sqrt{13} \tilde{x}_2 + \tilde{w}_2 \\ \tilde{y}_3 &= 2 \tilde{x}_3 + \tilde{w}_3 \end{aligned} \right\} \begin{array}{l} 3 \text{ Decoupled} \\ \text{channels.} \\ \text{Spatial} \\ \text{Multiplexing} \end{array}$$

Optimal Power Allocation:

↳ To maximize sum-rate

↳ Achieve Shannon Capacity

$$P_1 = \left(\frac{1}{\lambda} - \frac{\sigma^2}{\sigma_1^2} \right)^+ = \left(\frac{1}{\lambda} - \frac{\sigma^2}{52} \right)^+$$

$$P_2 = \left(\frac{1}{\lambda} - \frac{\sigma^2}{\sigma_2^2} \right)^+ = \left(\frac{1}{\lambda} - \frac{\sigma^2}{13} \right)^+$$

$$P_3 = \left(\frac{1}{\lambda} - \frac{\sigma^2}{\sigma_3^2} \right)^+ = \left(\frac{1}{\lambda} - \frac{\sigma^2}{4} \right)^+$$

Consider now noise power

$$\sigma^2 = 0 \text{ dB}$$

$$10 \log_{10} \sigma^2 = 0$$

$$\Rightarrow \sigma^2 = 10^{0/10} = 1$$

$$P = \text{Total Tx Power} = 3 \text{ dB}$$

$$10 \log_{10} P = 3$$

$$\Rightarrow P = 10^{3/10} = 10^{0.3} \approx 2$$

$$P_1 + P_2 + P_3 = 2$$

$$\left(\frac{1}{\lambda} - \frac{1}{52}\right) + \left(\frac{1}{\lambda} - \frac{1}{13}\right) + \left(\frac{1}{\lambda} - \frac{1}{4}\right) = 2$$

$$\Rightarrow \frac{1}{\lambda} = \frac{2 + \frac{1}{52} + \frac{1}{13} + \frac{1}{4}}{3}$$
$$= 0.7821$$

P_1, P_2, P_3 — Powers for Optimal Power Allocation

$$P_1 = 10 \log_{10} \left(\frac{1}{\lambda} - \frac{\sigma^2}{52} \right)$$

$$= 10 \log_{10} \left(0.7821 - \frac{1}{52} \right)$$

$$= \underline{-1.1755 \text{ dB}}$$

$$P_2 = 10 \log_{10} \left(0.7821 - \frac{1}{13} \right) = \underline{-1.517 \text{ dB}}$$

$$P_3 = 10 \log_{10} \left(0.7821 - \frac{1}{4} \right) = \underline{-2.74 \text{ dB}}$$

Power allocated decreases
as the gain σ_i of the
channel decreases.